FORT STREET HIGH SCHOOL

Name: ____________________

Teacher: ____________________

Class: ____________________

2016

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

Time allowed: 2 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
HE2, HE4	Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry, polynomials and circle geometry.	11, 12
HE3, HE5 HE6	Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	13
HE7	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	14

Total Marks 70

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 60 Marks
Attempt Questions 11-14,
Allow about 1 hour 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 60	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
	Percent	

General Instructions:

- Questions 11-14 are to be started in a new booklet.
- The marks allocated for each question are indicated.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used.

SECTION I (One mark each)

Answer each question by circling the letter for the correct alternative on this sheet.

1. Find $\frac{d}{dx}\left(\sin^{-1} 2x\right)$

(A) $\frac{2}{\sqrt{1-4x^2}}$

(B) $\frac{-2}{\sqrt{1-4x^2}}$

(C) $\frac{2}{\sqrt{1-2x^2}}$

(D) $\frac{-2}{\sqrt{1-2x^2}}$

2. What are the coordinates of the point that divides the interval joining the points $A(7,1)$ and $B(0,-6)$ internally in the ratio $4:3$?

(A) $(3,-3)$

(B) $(3,-2)$

(C) $(4,-2)$

(D) $(4,-3)$

3. Which of the following is an expression for $\int \cos^2 x \sin x \, dx$ using the substitution $u = \cos x$

(A) $2\cos x \sin x + c$

(B) $\cos^3 x + c$

(C) $\frac{1}{3}\cos^3 x + c$

(D) $-\frac{1}{3}\cos^3 x + c$

4. Find the exact value of $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$

(A) $\frac{1}{2}(\frac{\pi}{4} - \frac{1}{\sqrt{2}})$

(B) $\frac{\pi + 2}{8}$

(C) $\frac{1}{2}(\frac{\pi}{4} + \frac{1}{\sqrt{2}})$

(D) $\frac{\pi - 2\sqrt{2}}{8}$

5. How many numbers greater than 3000 can be formed with the digits 2, 3, 4 and 5 if no digit is used more than once in a number?

(A) 96
(B) 120
(C) 196
(D) 18

6. When $g(x)$ is divided by $x^2 + x - 6$ the remainder is $7x + 13$. What is the remainder when $g(x)$ is divided by $x + 3$?

(A) 55
(B) -5
(C) -8
(D) 34

7. What is the domain and range of $y = \sin^{-1}(\frac{x}{3})$?

(A) D: $-3 \le x \le 3$. R: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(B) D: $-\frac{2}{3} \le x \le \frac{2}{3}$. R: $0 \le y \le \pi$
(C) D: $-\frac{1}{3} \le x \le \frac{1}{3}$. R: $-\pi \le y \le \pi$
(D) D: $-1 \le x \le 1$. R: $-\pi \le y \le \pi$

8. If $t = \tan\frac{1}{2}\theta$ which of the following expressions is equivalent to $\sec\theta$?

(A) $\frac{1+t^2}{1-t^2}$
(B) $\frac{2t}{1+t^2}$
(C) $\frac{1-t^2}{1+t^2}$
(D) $\frac{2t}{1-t^2}$

9. Eden, Toby and four friends arrange themselves at random in a circle. How many arrangements are possible if Eden and Toby are **not** together?
(A) 20
(B) 96
(C) 72
(D) 119

10. A particle is moving with simple harmonic motion in a straight line so that its displacement $x\,cm$ from a fixed point 0 in the line at time t is defined by $x = 4\sin 2t$. Which of the following is the correct equation for v as a function of x?

(A) $v = \pm\sqrt{4(1-4x^2)}$

(B) $v = \pm\sqrt{4(16-x^2)}$

(C) $v = \pm\sqrt{4(1+4x^2)}$

(D) $v = \pm\sqrt{16(4-x^2)}$

SECTION II (15 marks each)

Answer each question in the appropriate booklet. Extra writing booklets are available.

Question 11: Use a separate writing booklet

(a) Evaluate $\lim_{x \to 0} \frac{\tan 4x}{x}$ **1**

(b) Find the acute angle between the lines $y = 2x - 5$ and $y = 6 - 3x$ **2**

(c) Solve $\frac{1}{x+1} \geq 1 - x$ **3**

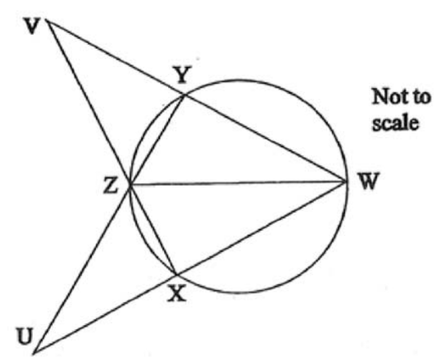
(d) $\int_0^{\frac{2}{5}} \frac{dx}{\sqrt{16 - 25x^2}}$ **3**

(e) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x = 2at, y = at^2$.

i. Find M, the midpoint of PQ **1**

ii. Show that, if the gradient of PQ is constant, the locus of M is a line parallel to the y-axis **2**

(f) In the diagram, UZY, XZV, VYW and UXW are all straight lines. Given ZW bisects $\angle XWY$ and $\angle WUZ = \angle WVZ$, prove that XW=YW. **3**



Question 12: Use a separate writing booklet

(a) Solve $\cos x - \sqrt{3}\sin x + 1 = 0$ for $0 \le x \le 2\pi$ **3**

(b)

i. Show that a solution of $x \ln x - 1 = 0$ lies between $x = 1$ and $x = 2$. **1**

ii. Using $x = 2$ as a first approximation, apply Newton's method once to obtain a better approximation. Give your answer to one decimal place. **2**

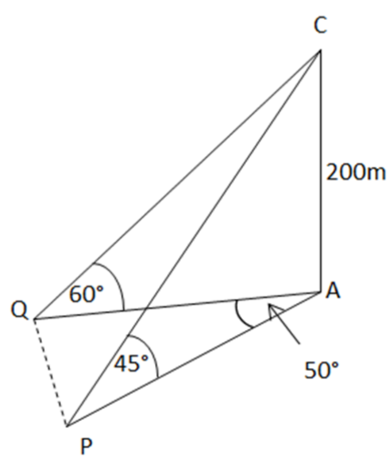
(c) A mixed tennis team consisting of 2 men and 2 women is to be chosen from 5 men and 7 women.

i. Find the probability that a particular woman is in the selected team **2**

ii. If one of the original 5 men is selected as the captain of the team, find the probability that his brother, who was one of the original 5 men, is also in the team. **1**

(d) Prove by Mathematical Induction that $5^{2n} - 1$ is divisible by 6 when n is a positive integer. **3**

(e) From the top, C, of a vertical cliff, 200 m high, two ships P and Q are observed at sea level. A is the foot of the cliff at sea level. P is due south of A and the angle of elevation of C from P is 45°. Q is S50°W of A and the angle of elevation of C from Q is 60°. Find the distance PQ (to the nearest metre) **3**



Question 13: Use a separate writing booklet

(a) The acceleration, a, of a particle is given in terms of its position, x, by the equation $a = 2x^3 + 2x$

i. If $v = 2$ when $x = 1$, show that $v^2 = (1 + x^2)^2$ **3**

ii. Show that, if $x = \frac{1}{\sqrt{3}}$ when $t = 0$, then $t = \frac{\pi}{6}$ when $x = \sqrt{3}$ **3**

(b) Given that the equation $x^3 + px^2 + qx + r = 0$ has a triple root, show that $pq = 9r$. **2**

(c) Sand pours onto the ground and forms a cone where the semi-vertical angle is $60°$. The height of the cone at time t seconds is h cm and the radius of the base is r cm. Sand is being poured onto the pile at a rate of $12cm^3 / s$. Find the rate at which the height is increasing at the instant when the height is 12 cm. **3**

(d) The rate at which a body cools in air is assumed to be proportional to the difference between its temperature T and the constant temperature S of the surrounding air. This can be expressed by the differential equation $\frac{dT}{dt} = k(T - S)$ where t is the time in hours and k is a constant.

i. Show that $T = S + Be^{kt}$, where B is a constant, is a solution of the differential equation. **2**

ii. A heated body cools from $80°C$ to $40°C$ in 2 hours. The air temperature S around the body is $20°C$. Find the temperature of the body after one further hour has elapsed. Give your answer correct to the nearest degree. **2**

Question 14: Use a separate writing booklet

(a) The speed v m/s of a point moving along the x-axis is given by $v^2 = 36 - 6x - 2x^2$ where x is in metres.

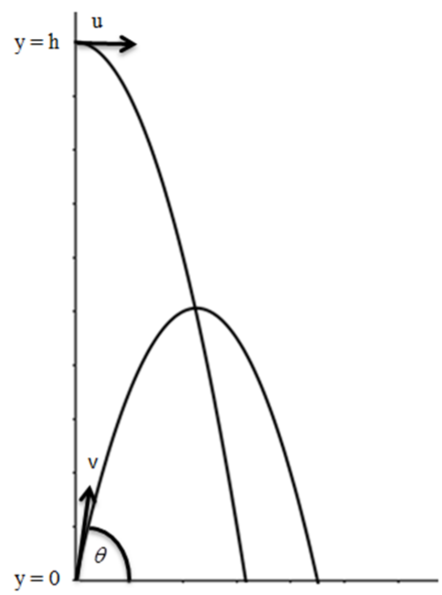
i. Prove that the motion is simple harmonic and find the centre of motion. **2**

ii. Find the period and amplitude of the motion. **2**

iii. Find the maximum speed (give your answer to 1 decimal place). **1**

(b) Find the general solutions of the equation $\sin 2\theta = \sin^2\theta$ **3**

(c) Two projectiles A and B are thrown simultaneously. B is thrown horizontally from the top of a building of height h, with velocity u, and A from the bottom of the same building with velocity, v at an angle θ to the horizontal.



Given the equations of motion for each projectile are:

Projectile A: $x_A = vt\cos\theta$ and $y_A = vt\sin\theta - \frac{1}{2}gt^2$

Projectile B: $x_B = ut$ and $y_B = h - \frac{1}{2}gt^2$

DO NOT PROVE THESE EQUATIONS OF MOTION!

i. Determine how u and v are related for a collision to take place. **1**

ii. Show that the height of the point of collision is given by $y = h - \dfrac{gh^2}{2v^2\sin^2\theta}$. **2**

iii. If the collision takes place at the vertex of Projectile A's trajectory, show that $\tan\theta = \dfrac{\sqrt{gh}}{u}$. **2**

[Note: the time it takes for Projectile A to reach its vertex is $t = \dfrac{v\sin\theta}{g}$]

iv. Show that the collision in part (ii) takes place half-way up the tower. **2**

2016 Mathematics Extension 1 Trial Solutions

1	$\frac{d}{dx}\sin^{-1}x = \frac{2}{\sqrt{1-4x^2}}$	A
2	$x = \frac{4\times 0+3\times 7}{4+3} = 3$ $y = \frac{4\times -6+3\times 1}{4+3} = -3$	A
3	$u = \cos x$ $\frac{du}{dx} = -\sin x$ $-\int u^2 du = -\frac{u^3}{3}+c = \frac{-\cos^3 x}{3}+c$	D
4	$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$ $\int_0^{\frac{\pi}{4}} \cos^2 x\ dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}\cos 2x + \frac{1}{2}$ $= \left[\frac{1}{4}\sin 2x + \frac{1}{2}\right]_0^{\frac{\pi}{4}}$ $= \left(\frac{1}{4}\sin 2\times\frac{\pi}{4}+\frac{1}{2}\right)-(0+0)$ $= \frac{1}{4}+\frac{\pi}{8} = \frac{2+\pi}{8}$	B
5	${}^3C_1 \times 3\times 2\times 1 = 18$	D
6	$g(x) = Q(x)(x^2+x-6)+(7x+13)$ $g(x) = Q(x)(x+3)(x-2)+(7x+13)$ Remainder when divided by $(x+3)$ is g(-3) $g(-3) = 0+7(-3)+13 = -8$	C
7	$D: -1 \le \frac{x}{3} \le 1$ *i.e.* $-3 \le x \le 3$ $R: -\frac{\pi}{2} \le y \le \frac{\pi}{2}$	A
8	$\cos\theta = \frac{1-t^2}{1+t^2}$ $\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1+t^2}{1-t^2}$	A
9	$5!-2\times 4! = 72$	C

10	$x = 4\sin 2t$ $v = \frac{dx}{dt} = 8\cos 2t$ $v^2 = 64\cos^2 2t$ $= 64(1-\sin^2 2t)$ $= 64\left(1-\frac{x^2}{16}\right)$ $= 4\left(16-x^2\right)$ $v = \pm\sqrt{4\left(16-x^2\right)}$ OR using the formula $v^2 = n^2(a^2-x^2) = 4(16-x^2)$ $v = \pm\sqrt{4(16-x^2)}$	B

Question 11

Marker's Comments

(a) $\lim_{x \to 0} \frac{\tan 4x}{x} = 4 \lim_{x \to 0} \frac{\tan 4x}{4x} = 4$ ❶ *for answer*

Generally well done.

(b) $\tan\theta = \left| \frac{2 - -3}{1 + 2 \times -3} \right|$ ❶

$= 1$

$\theta = \frac{\pi}{4}$ ❶

Some students used incorrect formula.

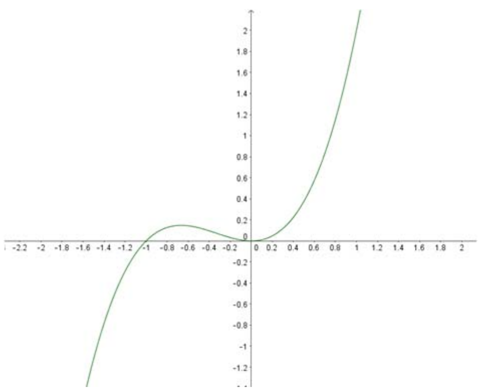
(c) $(1+x) \geq (1-x)(1+x)^2$

$(1+x) - (1-x)(1+x)^2 \geq 0$ ❶

$(1+x)(1-(1-x)(1+x)) \geq 0$

$(1+x)x^2 \geq 0$ ❶

$\therefore \; x = -1, 0$



$\therefore x > -1$ ❶ *for graph and answer*

Many students included -1 in their solutions not realising due to the original equation -1 can't be a solution.
A few people forgot to multiply both sides by $(1+x)^2$.

(d)

$$\int_0^{\frac{2}{5}} \frac{dx}{\sqrt{16 - 25x^2}} = \int_0^{\frac{2}{5}} \frac{dx}{5\sqrt{\frac{16}{25} - x^2}}$$ ❶

$$= \frac{1}{5}\left[\sin^{-1} \frac{x}{\frac{4}{5}} \right]_0^{\frac{2}{5}}$$ ❶

$$= \frac{1}{5}(\sin^{-1}\frac{1}{2} - \sin^{-1} 0) = \frac{1}{5} \times \frac{\pi}{6} = \frac{\pi}{30}$$ ❶

Generally well done.
Some students forgot to factorise out the $\frac{1}{5}$.

(e) i) $M = \dfrac{2ap+2aq}{2}, \dfrac{ap^2+aq^2}{2}$ $= ap+aq, \dfrac{ap^2+aq^2}{2}$ ❶ *for answer* ii) $m_{PQ} = \dfrac{aq^2-ap^2}{2aq-2ap} = \dfrac{p+q}{2} = k$, a constant ❶ For the point M $x = a(p+q) = 2ak$ ❶ Since a and k are constant, the locus M is a line parallel to the y-axis	Some students found the gradient of the chord but didn't recognise that they had to substitute into x. Others tried to differentiate and substitute into y.
(f) $\angle U = \angle V$ (*given*) $\angle UZX = \angle VZY$ (*vertically opposite angles*) *Now* $\angle ZXW = \angle UZX + \angle U$ (*exterior angle of a triangle*) *and* $\angle ZYW = \angle VZY + \angle V$ (*exterior angle of a triangle*) $\therefore \angle ZXW = \angle ZYW$ (*equal to sum of equal angles*) ❶ *In* ΔZXW *and* ΔYZW, ZW *is common* $\angle ZXW = \angle ZYW$ (*proven above*) $\angle XWZ = \angle YWZ$ (*given ZW bisects* $\angle YWX$) $\therefore \Delta XZW \equiv \Delta YZW$ (*AAS*) ❶ *and* $XW = YW$ (*matching sides in congruent* Δ'*s*) ❶	Most students were unable to write correct formal proofs.

Question 12 Marker's Comments

(a)

$$\cos x-\sqrt{3}\sin x = A\cos x\cos\theta - A\sin x\sin\theta = A\cos(x+\theta)$$

$$where\ \ A\cos\theta = 1\ \ and\ \ A\sin\theta = \sqrt{3}$$

$$Then\ \frac{A\sin\theta}{A\cos\theta} = \tan\theta = \sqrt{3},\ \ \theta = \frac{\pi}{3}$$

$$A = \sqrt{1^2+(\sqrt{3})^2} = 2$$

$$\therefore 2\cos\left(x+\frac{\pi}{3}\right)+1 = 0$$ ❶ *for R and* ❶ *for* α

$$\cos\left(x+\frac{\pi}{3}\right) = -\frac{1}{2}$$

$$x+\frac{\pi}{3} = \frac{2\pi}{3},\frac{4\pi}{3}$$

$$x = \frac{\pi}{3},\ \pi\quad for\ \ 0\le x\le 2\pi$$ ❶

Mostly well done.
Students left answers in general form.
Students should remember if using the t-formula to test for π.

(b)

i)

$f(1) = 1\times\ln 1 - 1 = -1\ \ which\ \ is < 0$

$f(2) = 2\times\ln 2 - 1 = 0.386...\ \ which\ \ is > 0$

Since $f(1) < 0$ and $f(2) > 0$ there is a solution of $x\ln x - 1$
between $x = 1$ and $x = 2$. ❶

Students did not specify that the root is due to a change in sign. A mark was not lost but students should try and explain their answers.

ii)

$f'(x) = \ln x + 1$ ❶

By Newton's method :

$$x_1 = x - \frac{f(x)}{f'(x)} = x - \frac{x\ln x - 1}{\ln x + 1}$$

$$if\ \ x = 2\ \ x_1 = 2 - \frac{2\ln 2 - 1}{\ln 2 + 1} = 1.77... \approx 1.8$$ ❶ *for answer*

Students differentiated incorrectly.

(c)

i) Probability of a particular woman

$$= \frac{\text{teams with a particular woman}}{\text{total number of possible teams}}$$

$$= \frac{{}^6C_1\times{}^5C_2}{{}^7C_2\times{}^5C_2} = \frac{60}{210} = \frac{2}{7}$$

❶ for numerator and ❶ for denominator

Not very well done.

ii) Probability of captain and brother

$$= \frac{\text{no. of teams with his brother}}{\text{number of possible teams}}$$

$$= \frac{{}^7C_2}{{}^7C_2 \times {}^4C_1} = \frac{21}{84} = \frac{1}{4}$$ ❶ *for answer*

(d)

Show true for $n = 1$

$5^2 - 1 = 24$ which is divisible by 6

Assume true for $n = k$

$5^{2k} - 1 = 6m$ where m is a positive integer

❶ *for show and assume steps*

Prove true for $n = k + 1$

$LHS = 5^{2(k+1)} - 1$

$= 5^{2k+2} - 1$

$= 5^{2k}5^2 - 1$

$= 25(5^{2k} - 1) - 1 + 25$ ❶

$= 25(6m) - 1 + 25$

$= 25(6m) + 24$

$= 6(25m + 4)$ ❶

Now m is an integer so $25m + 4$ is an integer

Since the statement is true for $n = k$, it is true for $n = k + 1$.

Hence the statement is true for all positive integers by the principal of Mathematical Induction.

Very well done.

(e)

$\tan 45 = \frac{200}{AP}$, $AP = \frac{200}{\tan 45}$

$\tan 60 = \frac{200}{AQ}$, $AQ = \frac{200}{\tan 60}$ ❶

$PQ^2 = AP^2 + AQ^2 - 2 \times AP \times AQ \times \cos 50$ ❶

$$PQ = \sqrt{\left(\frac{200}{\tan 45}\right)^2 + \left(\frac{200}{\tan 60}\right)^2 - 2 \times \frac{200}{\tan 45} \times \frac{200}{\tan 60} \times \cos 50}$$

$= 153.77...$

$= 154\ m$ ❶

Students did not get $\tan 45 = 1$.

Students used sine in the cosine rule.

Question 13 Marker's Comments

(a)

i)

$a = 2x^3 + 2x$

$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = 2x^3 + 2x$ ➊

$\frac{1}{2}v^2 = \frac{2x^4}{4} + \frac{2x^2}{2} + c$

$2 = \frac{1}{2} + 1 + c$

$\therefore c = \frac{1}{2}$

$\frac{1}{2}v^2 = \frac{x^4}{2} + x^2 + \frac{1}{2}$ ➊ for integration and constant

$\mathrm{v}^2 = x^4 + 2x^2 + 1 = \left(x^2 + 1\right)^2$ ➊

Mostly well done.

ii)

$v = \pm\left(x^2 + 1\right)$

$but\ \ v = 2\ (> 0)\ \ when\ \ x = 1$

$\therefore v = \left(x^2 + 1\right)$ ➊ with explanation for disregarding negative

$\frac{dt}{dx} = \frac{1}{x^2 + 1}$

$so\, t = \tan^{-1} x + c$ ➊

$Now\ \ x = \frac{1}{\sqrt{3}}\ \ when\ \ t = 0$

$\therefore c = -\tan^{-1}\frac{1}{\sqrt{3}} = -\frac{\pi}{6}$

$so\ \ t = \tan^{-1} x - \frac{\pi}{6}$

$when\ \ x = \sqrt{3},\ t = \tan^{-1}\sqrt{3} - \frac{\pi}{6} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ ➊

Many students did not justify their decision to disregard $v = -\left(x^2 + 1\right)$.

Some did not realise $\int \frac{1}{x^2 + 1} = \tan^{-1} x + c$.

(b)

$\alpha+\alpha+\alpha=3\alpha=-p....(1)$

$\alpha\alpha+\alpha\alpha+\alpha\alpha=3\alpha^2=q...(2)$

$\alpha\alpha\alpha=\alpha^3=-r...(3)$ ❶ for all three equations

$(1)\times(2)$

$9\alpha^3=-pq$

$9(-r)=-pq$ ❶

$\therefore\ pq=9r$

Some students did not realise a triple root means all three roots are identical.

(c)

$\frac{dh}{dt}=?$

$\frac{dV}{dt}=12cm^3/s$

$\tan 60=\frac{r}{h}$

$\therefore r=h\tan 60$ ❶

$V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi(h\tan 60)^2 h$

$=\frac{1}{3}\pi h^3\tan^2 60$

$=\pi h^3$

$\frac{dV}{dh}=3\pi h^2$ ❶

$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{1}{3\pi(12)^2}\times 12=\frac{1}{36\pi}cm/s$ ❶

Some students did not recall formula for the volume of a cone.

Some students did not obtain $r=h\tan 60$.

Some students did not replace r with $h\tan 60$ before differentiating.

(d)	
i)	
Differentiating $T = S + Be^{kt}$ to obtain $\frac{dT}{dt} = kBe^{kt}$ ....(1) ❶	Mostly well done
Re-arranging $T = S + Be^{kt}$ to obtain $Be^{kt} = T - S$	
and substituting into equation (1) ❶	
to obtain $\frac{dT}{dt} = k(T - S)$	
ii)	
$T = 80 \quad t = 0,\ T = 40 \quad t = 2,\ \ S = 20,\ \ T = ? \quad t = 3$	Mostly well done
$80 = 20 + Be^{0} \quad \therefore B = 60$	
$40 = 20 + 60e^{2k}$	
$e^{2k} = \frac{1}{3}$	
$2k = \ln\left(\frac{1}{3}\right), \quad k = \frac{1}{2}\ln\left(\frac{1}{3}\right)$ ❶ *for k*	
$t = 3, \quad T = 20 + 60e^{\frac{1}{2}\ln\left(\frac{1}{3}\right)\times 3} = 32°$ ❶	

Question 14

Marker's Comments

(a)

i) $\frac{1}{2}v^2 = 18 - 3x - x^2$

$$\frac{d\left(\frac{1}{2}v^2\right)}{dx} = -3 - 2x$$

$$= -1(3 + 2x)$$

$$= -2\left(x - \left(-\frac{3}{2}\right)\right)$$

$$= -n^2(x - b) \quad where\ b = -\frac{3}{2}$$ ❶

$C.O.M = -\frac{3}{2}$ ❶

To prove SHM, students needed to express acceleration in the form $\ddot{x} = -n^2(x - b)$. Many students could not do this. Also, centre of motion had to be stated here. Again, many students did not do this.

ii)

$T = \frac{2\pi}{n} = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ ❶

max amplitude occurs when $v = 0$

$0 = 18 - 3x - x^2$

$0 = (6 + x)(3 - x)$

$x = -6$ or $x = 3$

C.O.M is $-\frac{3}{2}$

so amplitude is $4.5m$ ❶

Generally well done. Errors were carried from (i) – this was considered. Note that amplitude is positive.

iii)

Maximum speed occurs when particle passes $x = -1.5$

$v^2 = 36 - 6(-1.5) - 2(-1.5)^2 = 40.5$

$\therefore$ max speed is 6.4 m/s

Speed = $|v|$. Generally well done.

(b)

$\sin 2\theta = \sin^2\theta$

$2\sin\theta\cos\theta = \sin^2\theta$

$\sin\theta(2\cos\theta - \sin\theta) = 0$ ❶

$\sin\theta = 0\ or\ 2\cos\theta - \sin\theta = 0$

$\therefore \sin\theta = 0\ or\ \tan\theta = 2$ ❶

$\therefore \theta = n\pi\ or\ \theta = n\pi + \tan^{-1} 2$ ❶

If students cancelled $\sin\theta$, only 1 mark was awarded. That is, first 2 marks were deducted. Do not mix radians and degrees as solution is incorrect.
Try to give the simplest general solutions!
Subsidiary angle method can be used to solve $2\cos\theta - \sin\theta = 0$ but too complicated.

(c)

i) $vt\cos\theta = ut$

$\therefore v\cos\theta = u$ ❶

Well done.

ii) $vt\sin\theta - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$

$t = \frac{h}{v\sin\theta}$ ❶

$y = h - \frac{1}{2}\left(g\left(\frac{h}{v\sin\theta}\right)^2\right) = h - \frac{gh^2}{2v^2\sin^2\theta}$ ❶

Generally, well done.

iii)

$t = \frac{v\sin\theta}{g}$

$\frac{h}{v\sin\theta} = \frac{v\sin\theta}{g}$ ❶

$h = \frac{v^2\sin^2\theta}{g}$

$gh = v^2\sin^2\theta$

$\sqrt{gh} = v\sin\theta$

$= \sin\theta \times \frac{u}{\cos\theta}$ ❶

$= u\tan\theta$

$\tan\theta = \frac{\sqrt{gh}}{u}$

Poorly done. Many students wrote pages of irrelevant working.

iv)

$y = h - \frac{gh^2}{2v^2\sin^2\theta}$

from part ii)

$gh = v^2\sin^2\theta$ ❶

$y = h - \frac{gh^2}{2gh} = \frac{h}{2}$ ❶

Same comment as for (iii). Also when proving $y = \frac{h}{2}$, do not combine the LHS and RHS in your proof, i.e. you cannot assume it's true to prove it.